An Introduction to the *Learning Progressions Frameworks Designed for Use with The Common Core State Standards in Mathematics K-12*

Karin K. Hess, NCIEA, Project Director/Principal Author & Jacqui Kearns, NAAC at UKY, NAAC Principal Investigator

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Developing the Learning Progressions Frameworks

Two separate committees worked on this project during 2010 in each content area (mathematics, language arts, and science). Educators represented seventeen (17) different states, eight (8) colleges and universities, and seven (7) state or national educational organizations. The first committee to meet was comprised of content experts and researchers from both general education and special education. Their tasks were to review and synthesize the research literature about mathematics learning and draft the conceptual **learning progressions frameworks** (LPFs), in this case for mathematics. This work included identification of **enduring understandings** and essential **learning targets** for the elementary (K-4), middle (5-8), and high school (9-12) grade spans. The second committee included a mix of master teachers and professional development providers with classroom experience at each grade span – organized in teams of both general education and special education working together. Curriculum development committee tasks were to: (1) "zoom in" and break down specific targeted sections of the draft LPFs into what we called more detailed "**mini progressions**" for a smaller grade span, often adding some additional "interim steps" (**progress indicators**) to the mini progressions; (2) use the more detailed and focused mini progressions to design instructional modules (with a series of 4-6 detailed lessons) illustrating how a teacher in the general education classroom might move students along this smaller grain-sized learning progression using best practices in instruction; and (3) draw from best practices in instruction for students with significant cognitive disabilities to incorporate suggestions to each lesson plan for how to make the academic content more accessible for *all* students.

The approach used to identify the content progressions and specific standards within the *Common Core State Standards* (CCSS) considered three important dimensions. First, national content experts and researchers in mathematics were asked to identify specific content strands that represented a way to organize essential learning for all students, K-12. Next, the committee was asked to describe the "enduring understandings" (as defined by Wiggins and McTighe, 2005) for each particular content strand, as well as articulate what the learning targets would look like if students were demonstrating achievement of the enduring understandings at the end of each grade span (K-4, 5-8, and 9-12). The grade span **learning targets** for each strand are stated as broader performance indicators (e.g., use equations and expressions involving basic operations to represent a given context; Build flexibility with whole numbers and fractions to understand the nature of number and number systems). The larger grained grade span learning targets are designed to describe progressively more complex demonstrations of learning across the elementary to high school grade spans and use wording similar to what one might see in performance level descriptors for a given grade or grade span.

In mathematics, **six major LPF strands** were established. It is not the intent that skills/concepts from a particular strand be taught in isolation, or in a linear sequence, but rather be integrated among strands, such as in a problem solving situation where students are demonstrating their

understanding of measurement concepts while applying their knowledge of numbers and operations and using symbolic expression. *In other words, the LPFs should be thought of as a general map for learning, not a single route to a destination.*

- **Symbolic Expression (SE)**
- **The Nature of Numbers & Operations (NO)**
- **Measurement (ME)**
- **Patterns, Relations, & Functions (PFR)**
- **Geometry (GM)**
- **Data Analysis, Probability, & Statistics (DPS)**

These first two steps - developing six major content strands, each with progressively more sophisticated or complex grade span learning targets - established the underlying conceptual framework that could be built upon across the grades and linked to specific research-based progressions of skills and concepts needed to achieve the designated learning targets.

Once the content committee had established the broader grade span learning targets for each strand, they were asked to identify and describe the essential skills and concepts needed to achieve the grade span expectations; use research syntheses to establish *a general order of how those skills and concepts emerge for most students*; and further break down the descriptors into smaller grades spans: K-2, 3-4, 5-6, 7-8, and high school. The descriptors of related skills and concepts became what we now call the **progress indicators** and the ordering/numbering used (1a, 1b, 1c, etc.) reflects the research base used to establish a general learning continuum. Descriptions of earlier skills build the foundation for later skills at the next grade level or grade span.

The final step in the LPF development process was to look backward and forward (grades K-12) to identify alignment with specific CCSS mathematics content standards in order to create guidance for a cohesive curriculum experience across grades. Sometimes multiple standards from within the smaller grade spans could be linked to the same progress indicator (PI); sometimes there was only one or no standard that aligned. For example, in some strands and grade spans you will see PI descriptors that do not link (align) with an existing CCSS standard; however, the research review identified critical learning needed at certain stages during the learning process or skills that may be essential for conceptual understanding and for making progress; therefore, progress indicators with no CCSS links are also included in the LPF to guide instruction and progress monitoring.

Alignment to the *Common Core State Standards*

Progress indicators (PIs) describe *observable learning* along the learning continuum for each strand in the learning progressions frameworks. While links between the LPF and most (83%-100% depending on the grade level)* of the *Common Core State Standards* (CCSS) in mathematics have been identified, the LPF also includes some descriptions of essential learning for which there may not be specific CCSS standards. Additionally, there are cases where a CCSS standard is linked to more than one progress indicator (in different mathematics strands and/or at multiple grade levels), and places where only part of the CCSS standard links to the progress indicator. This approach to alignment serves to focus a greater emphasis on how to interpret a student's learning path than on everything described in a particular standard. (The following pages illustrate an example of this alignment coding and how to "unpack" the LPF strands.)

*In high school mathematics, the content standards indicated with a (+) were not considered or linked with the LPF because of the asterisks in the CCSS document annotating the purpose for those standards as being additional: "Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example: (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards without a (+) symbol may also appear in courses intended for all students" (CCSS, p. 57, June 2010).

A conceptual view of learning progressions (Hess, 2008) is one of overlapping learning zones along a continuum of learning. At the lower end of the progression are "Novice" learners (at any grade level), who may (or may not) demonstrate the necessary prerequisite skills and concepts needed that can be built upon over time. A starting point for learning can be established, perhaps with a short pre-assessment or diagnostic assessment. Guided, targeted, and scaffolded practice can be employed to develop subsets of understanding (skills/concepts broken into smaller manageable and meaningful learning chunks). Later during the instructional cycle, instruction targets students' ability to develop schemas to organize and connect new learning and work more independently.
(The Zone of Proximal Development/ZPD is the range of potential each person has for learning at any given time, Vygotsky, 1978).

Sample Strand 1: Symbolic Expression

The statement of enduring understanding across all grades states WHY the learning is important.

Different learning targets show a progression from one grade span to the next grade span.

Symbolic Expression (SE): The use and manipulation of symbols and expressions provide a variety of representations for solving problems and expressing mathematical concepts, relationships, and reasoning.		
(K-4) Elementary School Learning Targets	**(5-8) Middle School Learning Targets**	**(9-12) High School Learning Targets**
E.SE-1 Use equations and expressions involving basic operations to represent a given context • *Represent numerical relationships using combinations of symbols (=, >, <) and numbers to form expressions and equations* • *Solve for unknown in simple number binary number sentences (e.g., ____ + 4 = 7);* • *Write equations showing inverse operations and related operations (e.g., addition-multiplication).*	*M.SE-1 Represent relationships and interpret expressions and equations in terms of a given context for determining an unknown value.* • *Represent mathematical relationships symbolically and solve for any variable (for 1st degree equations and for common formulas (literal equation);* • *Explain how to manipulate an algebraic expression to create equivalent expressions and provide step-by-step explanations and justifications.*	*H.SE-1 Represent relationships and interpret expressions and equations in terms of a given context (including complex and families of functions) for determining unknown values (including two or more variables).* • *Represent and interpret multi-step problems;* • *Represent complex numbers and vectors;* • *Demonstrate the relationship between systems of equations and matrix representations;* • *Represent the relationship between functions and modeling.*

Some Key Research Ideas Considered during Development - Symbolic Expression

Preschoolers who can count to ten (by rote) may not necessarily "know" the meanings of words beyond two, or three, or four; so the use of a number word need not guarantee comprehending a link to a given quantity (e.g., Huang, Spelke, & Snedekar, 2010).

Describing that symbols correspond to specific quantities (match symbol to set of specific quantities, etc.) is a necessary precursor that, if absent, renders the rest of the skills potentially meaningless rote procedural knowledge unlinked to conceptual understanding; research is demonstrating that this link is not present in all children to the extent adults assume it will be.

When children have a poor number 'sense,' the association between a symbol and a quantity may not be so obvious. Then, even for those who recognize this connection, the link between the two may not be automatic. Indeed Girelli and colleagues (2000) demonstrated that in typically achieving children this automaticity is not fully established until grade 2 or 3. This may continue to be an issue for a much longer time for a subset of individuals. Moreover, for primary school children with math difficulties, transcoding of written numerals is also less automatic (van Loosbroek, Dirkx, Hulstijn, & Janssen, 2009).

There is evidence that both the ability to rapidly represent non-symbolic quantities and the ability to map a number word to a quantity, contribute independently to math performance, even through middle school (Mazzocco, Feigenson, & Halberda, in press).

Arithmetic and algebra use the same symbols and signs but apply and interpret them differently. This can be very confusing to students particularly if their arithmetic concepts are weak. (Bamberger, Oberdorf, & Schultz-Ferrel, 2010, p. 69)

Reading and Interpreting the LPF for a Grade Span

NO: Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide use of computational strategies and algorithms.

The statement of enduring understanding across grade spans states WHY learning the skills and concepts (and standards) listed below are important and how they are generally applied.

(K-4) Elementary School Learning Targets

E.NO-2 Build an understanding of computational strategies and algorithms:

- *Fluently add, subtract, multiply, divide, and estimate;*
- *Perform and represent operations with whole numbers, fractions, and mixed numbers;*
- *Identify multiples and factors of whole numbers.*

K-4 Elementary School Grade Span Learning Targets

- By the end of grade 4, students demonstrate and apply the skills and concepts related to Numbers & Operations in a variety of situations or problem solving contexts.
- Learning targets are the more general/broad performance descriptors associated with specific skills and concepts at each grade level.

Grades K-2	Grades 3-4
Build understanding and fluency with operations...	Build understanding and fluency with operations...
E.NO.2a representing addition and subtraction in multiple ways (composing/decomposing numbers, diagrams, using objects, arrays, equations, number lines), including regrouping K.OA-1, 2, 3, 4; K.NBT-1 1.OA-1, 2, 5, 6; 1.NBT-4, 5, 6 2.OA-1, 4; 2.NBT-7	E.NO.2d modeling multiplication (equal-sized groups, arrays, area models, equal-sized jumps on number lines, multiplicative comparisons) and division (successive subtraction, partitioning, sharing) of whole numbers 3.OA-1, 2, 3, 4, 5 4.OA-1, 2, 3; 4.NBT-4, 5, 6
E.NO.2b explaining or modeling the relationship between addition and subtraction 1.OA-3, 4 2.NBT-5, 7, 9	E.NO.2e describing relationships between addition-multiplication; multiplication-division; addition-subtraction; why commutativity does not apply to subtraction or division 3.OA-7, 9; 3.NBT-2 4.OA-2, 4
E.NO.2c working flexibly with common addition and subtraction situations 1. OA-3, 5, 6, 8 2.OA-1, 2; 2.NBT- 2, 5, 7	E.NO.2f identifying factors and multiples of numbers 3.OA-6 4.OA-4
	E.NO.2g recognizing fractions as one number/one quantity, rather than two numbers (numerator and denominator) and using number lines to represent magnitude of fractions 3.NF-1, 2, 3a, 3c
	E.NO.2h adding, subtracting, and multiplying fractions, including mixed numbers 4-NF-3, 4

Text in blue denotes links to CCSS standards:
2.OA-1,2 means grade 2, Operations & Algebraic Thinking, standards 1 and 2 (See p. 19 of CCSS for mathematics)

Larger grade spans are then broken into smaller grade spans

What you see articulated in this LPF strand:

- "E" denotes all Elementary (K-4) progress indicators.
- Most LPF descriptors/progress indicators are stated in a more general way (e.g., using many related strategies; using both addition and subtraction) than a single CCSS standard; therefore progress indicators (PIs) often align with several CCSS standards at different grade levels within the grade span. This multi-standard alignment can provide insights into potential "mini progressions" for lesson design.
- Numerous CCSS standards align with the first descriptor under K-2 and can be interpreted that this progress indicator embodies many important foundational skills for all three grade levels, K, 1, and 2. Teachers at all of these grades may need to revisit lower grade level skills (and standards) for students needing reinforcement and some extra work on prerequisite skills.
- K students would spend most of their school year working on CCSS standards: K.OA-1, 2, 3, 4; and K.NBT-1 (linked to the first PI), while grades 1 and 2 would be addressing all three PIs and the associated CCSS standards in this general/a-b-c order.

"Unpacking" the LPF Grade Span for a Grade Level: In this second expanded LPF example, we illustrate how to "unpack" the LPF by grade level.

Data Analysis, Probability, and Statistics (DPS): **Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions.**

Elementary School Learning Targets

E.DPS-1 *Gather and interpret data to answer questions related to a particular/single context.*

- *Formulate questions, gather data, and build representations;*
- *Identify and describe variation in data, and describe and compare shapes of distributions and measures of central tendency.*

by Grade 4, all students should demonstrate these learning targets, applying skills and concepts for DSP-1.

Elementary Progress Indicators for Grades K-2	Grade K	Grade 1	Grade 2
Gather, organize, and interpret data by…	**Gather, organize, and interpret data by…**	**Gather, organize, and interpret data by…**	**Gather, organize, and interpret data by…**
E.DPS.1a posing questions of interest that can be answered by counting/collecting data (e.g., concrete comparisons about students, classroom materials, science topics) with teacher guidance **E.DPS.1b** identifying and sorting data/attributes; identifying rules for classifying data/attributes **E.DPS.1c** collecting and organizing/ representing data (e.g., picture graphs, tally charts, bar graphs) **E.DPS.1d** recognizing that data can take on different values **E.DPS.1e** describing and comparing data and beginning to identify what the data do or do not show (e.g., bar graphs, line plots, picture graphs) Progress Indicators describe concepts and skills along the learning continuum for the K-2 grade span. These skills & concepts build towards successful demonstration of learning targets for grade 4. The suggested order (a, b, c, etc.) is based on a review of	**E.DPS.1a** posing questions of interest that can be answered by counting/collecting data (e.g., concrete comparisons about students, classroom materials, science topics) with teacher guidance **K.CC-5, 6** **E.DPS.1b** identifying and sorting data/attributes; identifying rules for classifying data/attributes **K.MD-1, 2, 3; K.G-2, 4** **E.DPS.1c** collecting and organizing/ representing data (e.g., picture graphs, tally charts, bar graphs) **K.CC-5; K.MD-1, 2, 3** Kindergarten students who have demonstrated understanding of Progress Indicators E.DPS a, b, and c (and related K standards) will benefit from new experiences with the next two PIs as well as related grade 1 CCSS standards. **E.DPS.1d** recognizing that data can take on different values **E.DPS.1e** describing and comparing data and beginning to identify what the data do or do not show (e.g., bar graphs, line plots, picture graphs)	**E.DPS.1a** posing questions of interest that can be answered by counting/collecting data (e.g., concrete comparisons about students, classroom materials, science topics) with teacher guidance **K.CC-5, 6 (*may need to revisit*)** **1-MD-1** **E.DPS.1b** identifying and sorting data/attributes; identifying rules for classifying data/attributes **K.MD-1, 2, 3; K.G-2, 4 (*may need to revisit*)** **1.MD-1, 4; 1.G-1** **E.DPS.1c** collecting and organizing/ representing data (e.g., picture graphs, tally charts, bar graphs) **K.CC-5, 6; K.MD-1, 2, 3 (*may need to revisit*)** **1-MD-1, 4** **E.DPS.1d** recognizing that data can take on different values **1.MD-4** **E.DPS.1e** describing and comparing data and beginning to identify what the data do or do not show (e.g., bar graphs, line plots, picture graphs) **1.MD-4**	**E.DPS.1a** posing questions of interest that can be answered by counting/collecting data (e.g., concrete comparisons about students, classroom materials, science topics) with teacher guidance **K.CC-5, 6 (*may need to revisit*)** **1-MD-1 (*may need to revisit*)** **2.MD-2, 5, 9** **E.DPS.1b** identifying and sorting data/attributes; identifying rules for classifying data/attributes **K.MD-1, 2, 3; K.G-2, 4 (*may need to revisit*)** **1.MD-1, 4; 1.G-1 (*may need to revisit*)** **2.G-1** **E.DPS.1c** collecting and organizing/ representing data (e.g., picture graphs, tally charts, bar graphs) **K.CC-5, 6; K.MD-1, 2, 3 (*may need to revisit*)** **1-MD-1, 4 (*may need to revisit*)** **2.MD-1, 2, 4, 5, 6, 9, 10** **E.DPS.1d** recognizing that data can take on different values **1.MD-4 (*may need to revisit*)** **2.MD- 3, 9, 10** **E.DPS.1e** describing and comparing data and beginning to identify what the data do or do not show (e.g., bar graphs, line plots, picture graphs) **1.MD-4 (*may need to revisit*)** **2.MD-2, 4, 5, 6, 9, 10**

empirical research. If CCSS standards align with the Progress Indicators (PIs), they are listed below (in blue) and include each related CCSS standard in the grade span.	The first grade level in the grade span builds the foundation. If there are no CCSS aligned standards for PIs listed, students still need to have learning experiences for each PI.	At grade 1, students who have not built a solid foundation, may need to revisit or have added practice with selected PIs (and lower grade level CCSS standards) before going on.	The highest grade level (gr 2) in the grade span can have more intermediate steps in the LP than grades K or 1. PIs (and lower grade level CCSS standards) may need to be revisited for students who need additional reinforcement.

Expanded version of the LPF to show smaller learning progressions and links to the CCSS standards for the K-2 grade span

Elementary School Learning Targets

Data Analysis, Probability, and Statistics (DPS)- *DPS-1 Gather and interpret data to answer questions related to a particular/single context. Formulate questions, gather data, and build representations; Identify and describe variation in data, and describe and compare shapes of distributions and measures of central tendency.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.DPS.1a posing questions of interest that can be answered by counting/collecting data (e.g., concrete comparisons about students, classroom materials, science topics) *with teacher guidance* Highlighting indicates links among Progress Indicator & one or more CCSS standard	K.CC-5 5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. K.CC-6 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] ([1] Include groups with up to ten objects.)	1-MD-1 1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	2.MD-2 2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. 2.MD- 5 5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. 2.MD-9 9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
E.DPS.1b identifying and sorting data/attributes; identifying rules for classifying data/attributes Highlighting in 2 colors indicates two possible smaller LPs for instruction & assessment for this PI	K.MD-1 1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. K.MD-2 2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* K.MD-3 3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[1] K.G-2	1.MD-1 1. Order three objects by length; compare the lengths of two objects indirectly by using a third object. 1.MD-4 4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. 1.G-1 1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw	2.G-1 1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. 2.MD-10 10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.

	2. Correctly name shapes regardless of their orientations or overall size. **K.G-4** **4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts & attributes**	**shapes to possess defining attributes.**	